Wooden Shelves – Understanding why they sag

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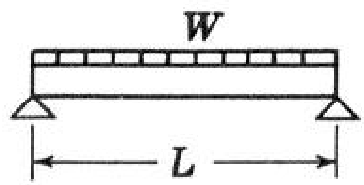
Why do shelves seem to sag so readily in the middle? If they are too thin or too long, then nature is against you even if you have the most exotic hard wood. How much thicker should you make them?

Well, those of us that studied mechanical engineering design may recall an equation that predicts what will happen – a bending equation that will help you to understand the effects of changes in dimensions.

The diagram shows a shelf supported loosely at both ends, and loaded with weight uniformly distributed across the length – a typical application. The shelf is rectangular in cross section, so the theory predicts a deflection (sag) δ in the middle according to this equation:



$$\delta = \frac{WL^3}{6.4Ebh^3}$$

Where: W is the distributed load
L is the length of the shelf
E is the modulus of elasticity (a measure of how strong the material is)
b is the width of the shelf
h is the height of the shelf

Absolute values are not important. This equation is to help you understand the effects of changing different values:

Logically, if you increase the load W, the deflection δ increases proportionally.

Also if you increase the length, the deflection increases, but to the third power of the length. So if you double the length, the deflection increases eight times! (2^3) This equation tells you why long shelves are bad news. Even a small reduction in length will dramatically reduce the deflection (sag).

However, usually the length is given by the design, as well as the width b. The properties of the material can make a difference too, but not much, as E only varies by a factor of 2 to 3 over a whole range of woods.

The only other variable left to play with is the thickness of the shelf, and it turns out that this has a dramatic effect. Logically, the deflection or bending decreases as you make the shelf thicker In this case the equation predicts that it will improve by the third power! This means that if you double the thickness, the droop goes down by a factor of $1/2^3 = 1/8$! So it is easy to make a large improvement by only a small increase in thickness. To halve the sag of a particular shelf, you need to increase the thickness by the cube root of 2 ($2^{1/3}$) which is 1.26. So an increase of 26% in the thickness will halve the droop in the shelf for the same load. This is the same as going from 16mm to 21 mm thickness.

I skipped over the effect of E which depends on the material properties. It is tempting to treat wood as a uniform, homogenous material, but it isn't. In the above example, it will be stiffer if the grain runs longitudinally, and much weaker if the grain runs across, which you can verify experimentally. This is why chipboard is much weaker than the same cross section of wood with the grain running in the correct direction. The grain in chipboard is random and the length of the individual fibres is short, so it is not able to resist bending nearly as well. Also if it should get wet, then you have a disaster.

(If you wish to make some real calculations, the Encyclopaedia of Wood published by the American Forest Products Laboratory and Sterling, 1987 gives tables of E for different species. E is quite variable and depends on the direction of the grain - working values of 1 to 2 million psi are a start.)